Contents

Validation of high-resolution global geopotential models over Sri Lanka using ground gravity & GPS-levelling data

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ABSTRACT

With the advent of new gravity-dedicated satellite missions (CHAMP, GRACE, and GOCE), the long wavelength gravity field modelling of the Earth is possible with remarkable accuracy. Numerous global geopotential models (GGMs) have been developed to date with improved satellite gravity information, enhanced land gravity and satellite altimeter data. In the recent past, high-resolution GGMs (HR-GGM) have been developed with significant accuracy. They can be useful for local and regional geodetic and geophysical applications, especially in areas with a lack of ground gravity data coverage. Inaccuracy analysis of GGMs is vital before using them in geodetic or geophysical applications. In this study, five HR-GGMs are evaluated against the absolute gravity, Bouguer anomaly and GPS-levelling data in Sri Lanka. Two regions with flat and rugged terrain, Jaffna and Bandarawela, were utilized to investigate their variations appropriately. Analysis of gravity and Bouguer anomaly revealed that even high-resolution global models are not capable of representing features in rugged mountainous areas because of the omission errors resulted due to the truncation of the model's gravity field at its maximum degree and order, but fitted quite well with flat terrain. A clear bias around 1.6 m of Sri Lankan GPS-levelling datum can be seen through the results of geoid height analysis of high mean values and comparatively low standard deviations. Overall, the recently released SGGUGM-2 model shows a better agreement with ground gravity and GPS-levelling data in Sri Lanka.

Keywords: Global geopotential models, Gravity data, GPS-levelling data.

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INTRODUCTION

The mass distribution of the Earth is not homogeneous. It comprises surface features such as mountains, plains, rivers, oceans, valleys, and trenches as well as sub-surface or internal mass structures like core and mantle. The inhomogeneity of the Earth's structure causes the variation of its gravity, which is a very useful tool, especially for disclosing internal mass structures. Also, gravity data can be used to explain various engineering and environmental problems such as determining surface or near-surface soil layer thickness and its moisture content (Niu*et al.,* 2007) and detection of buried tunnels, caves and sinkholes (Kaufmann *et al.*, 2011). In addition, on a global scale, understanding the details of the Earth's gravity field is crucial in satellite navigation and its applications in military missions. From a geodetic point of view, one of the fundamental applications of Earth's gravity is to determine the shape of its surface. The most reasonable figure of the Earth is defined as the geoid. According to Gauss-Listing's definition, the geoid is defined as "the equipotential surface of the Earth's gravity field" which coincides with the mean sea level of the oceans. "Averaging the ocean surface over time (at least over one year) or modelling ocean tides provides mean sea levels (MSL) for the corresponding time interval" (Torge, 2001). According to C.F. Gauss, the geoid is the "mathematical figure of the Earth" (Heiskanen and Moritz, 1967) though it is difficult to determine the exact analytical expression of the geoid. At present, precise geoid determination has become more important due to the development of Global Navigation Satellite Systems (GNSS). With the GNSS and space/air-borne radar systems (satellite altimetry, LIDAR and SAR), the capability of obtaining horizontal and vertical positions at any point on land or sea has significantly been improved. Therefore, the geoid height (N) and the ellipsoidal height derived from GNSS measurements (h) can be used to obtain the orthometric height (H) using Eq. (1), simply called the MSL height, without the sprit-levelling process, which is exhausting and time-consuming.

$$(h = H + N) \tag{1}$$

The establishment of precise local/regional geoids has been an important geodetic task for many national or regional surveying agencies (Smith and Milbert, 1999; Vergos and Sideris, 2002; Véronneau and Huang, 2007). One of the conventional methods for precise geoid determination is to use gravity observation through well-known Stokes integration with a remove-restore technique. In this approach, the geoid undulations due to long wavelength gravity field variations (N_{GGM}) are estimated by a Global Geopotential Model (GGM) and the short wavelength parts (N_T) are obtained from a regional topographic model. The remaining medium wavelength features of geoid undulation $(N_{\Delta g})$ are estimated from regional residual gravity anomalies obtained by removing the long wavelength and short wavelength components from the observed gravity anomalies over the region, with Stokes integration. The total geoid is given by:

$$N = N_{GGM} + N_{\Delta g} + N_T \tag{2}$$

The long wavelength component of N, N_{GGM} derived from GGM is given by

$$N_{GGM}(r,\varphi,\lambda) = \frac{GM}{r\gamma} \sum_{n=2}^{n\max} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos\varphi) \tag{3}$$

where (r,φ,λ) are the spherical coordinates of the computation point. The scaling parameters GM and a are the geopotential constant and semi-major axis of the reference ellipsoid. $\bar{P}_{nm}$ are the fully normalized associated Legendre functions. $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully-normalized, unit-less spherical harmonic model coefficients and γ is the normal gravity on reference ellipsoid.

In the recent past, high-resolution GGMs (HR-GGM) have been developed with significant accuracy. EGM2008 represents the first state-of-the-art HR-GGM that completes spherical harmonic degree and order 2160, and provides some additional coefficients up to degree 2190 (Pavlis et al., 2012). These HR-GGMs represent gravity field quantities with a wavelength of approximately 10 arc minutes, which equates to the spatial resolution of 5 arc minutes, depending on the latitude. Hence, any gravity field quantities with a spatial scale larger than 5 arc minutes are supposed to be represented by these models. Sri Lanka is a country with limited ground gravity data coverage is publicly available. Therefore, the accuracy of gravity field quantities provided by HR-GGMs over Sri Lanka is vital for gravity-related research and findings. In previous studies, an accuracy analysis of the global model has been performed using the BGI (International Gravimetric Bureau) gravity data (Prasanna *et al.*, 2021). This research mainly attempts to evaluate the HR-GGMs using ground gravity and GPS-levelling data.

High Resolution GGMs (HR-GGM)

With the advent of the gravity-dedicated satellite missions [CHAMP (in 2000), GRACE (in 2002) and GOCE (in 2009)], the long wavelength gravity field modelling of the Earth is possible with remarkable accuracy. Numerous GGMs have been developed to date with these improved satellite gravity information, enhanced land gravity and satellite altimeter data. All developed GGMs to date are available at *http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html*.

The first HR-GGM was the Earth Gravitational Model 2008 (EGM2008) which was publicly released by the U.S. National Geospatial-Intelligence Agency (NGA) EGM Development Team in April 2008 (Pavlis *et al.*, 2008, 2012: *http://earth-info.nga.mil /GandG/wgs84/gravitymod/egm2008/ index.html*). The development of this model is a major achievement in global gravity field modelling. It completes spherical harmonic degree and order 2160, and provides some additional coefficients up to degree 2190. These represent gravity field quantities with a wavelength of approximately 10 arc minutes ($\lambda = 360 / n_{\max}^{EGM}$ ≈ 10 arc minutes), which equates to the spatial resolution of 5 arc minutes ($\Delta x = 180 / n_{\max}^{EGM}$ ≈ 5 arc minutes ; 9 km, depending on the latitude). Hence, any gravity field quantities with a spatial scale larger than 5 arc minutes are supposed to be represented by this model. A comprehensive description of the model formation by integrating different data types (for example, satellite gravity, area-mean 5 arc minutes terrestrial free-air gravity, satellite altimetry, *etc*.) and methodologies have been given by Pavlis *et al.* (2012).

Since then, there were many HR-GGMs have been developed, such as EIGEN-6C4 (2014); GECO (2016), SGG-UGM-1 (2018); SGG-UGM-2 (2020), *etc*. These GGMs represent gravity field quantities with a wavelength of approximately 10 arc minutes, which equates to the spatial resolution of 5 arc minutes, depending on the latitude. Hence, any gravity field quantities with a spatial scale larger than 5 arc minutes are supposed to be represented by these models.

Study Area and Data Used

Two study areas in Sri Lanka: Jaffna and Bandarawela, were used for this analysis representing flat and rugged terrain (Figure 1). The land gravity data were recently observed in both regions using a Scintrex CG-6 gravity meter which is the state-of-the-art technology of modern gravity observations. It has a worldwide measurement range of over 8000 m Gals and a reading resolution of 0.0001 m Gal. In Jaffna and Bandarawela, 267 and 171 observed gravity points were used for the analysis, respectively. Four types of corrections: temperature, tilt, tide and drift were applied to the observed gravity, and the Shuttle Radar Topography Mission, SRTM (Farr *et al.*, 2007) global digital elevation model with one arc-second resolution was used for terrain corrections and calculation of Bouguer gravity anomalies. In order to analyze the geoid heights, 22 observed geoid heights at fundamental benchmarks (FBM) in Sri Lanka were used.

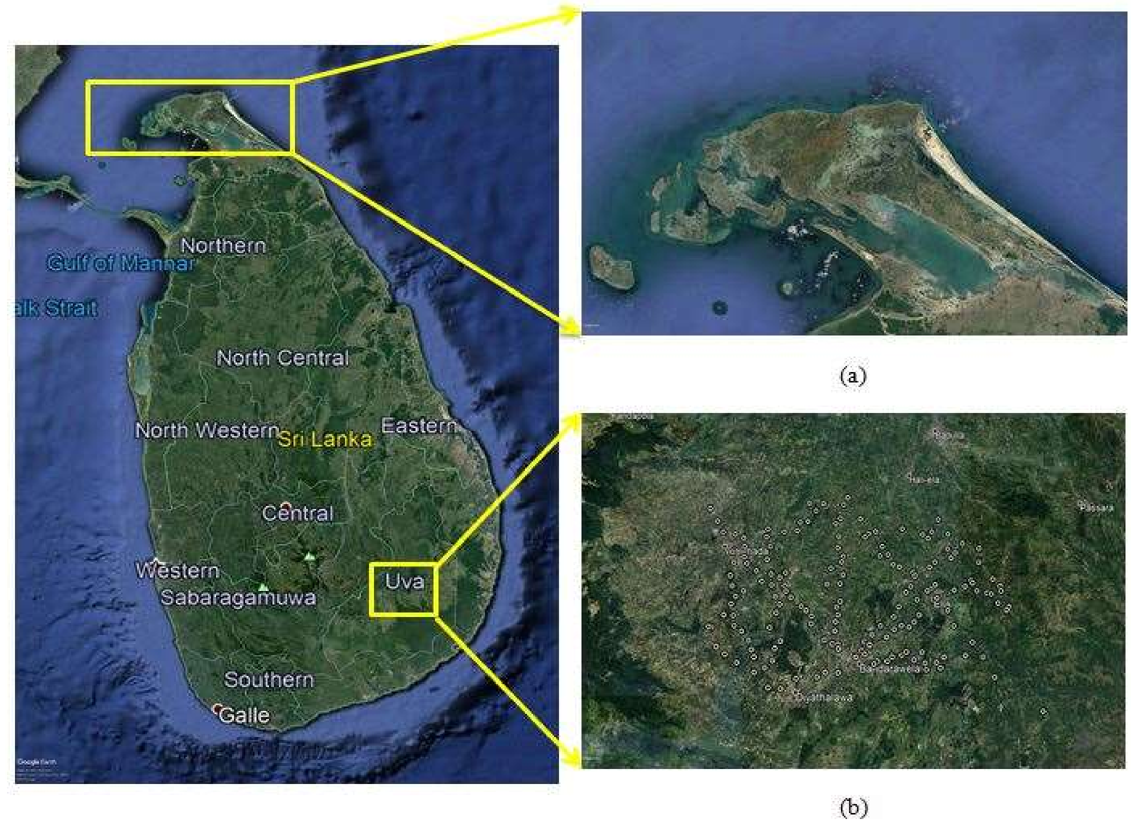

Figure 1: Study area (a) Jaffna (b) Bandarawela with observed stations

METHODOLOGY

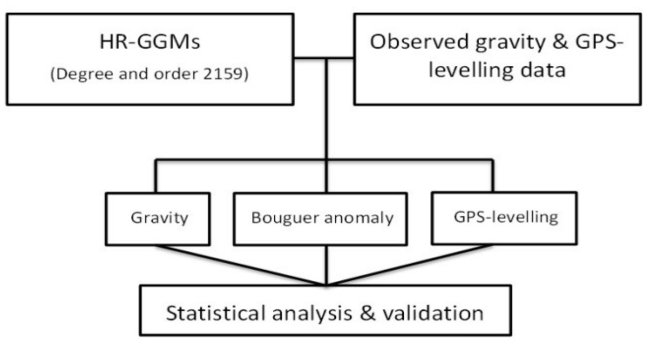

Figure 2: Computation procedure

For this study, five HR-GGMs which have been released so far were utilized: EGM2008 (2008); EIGEN-6C4 (2014); GECO (2016), SGG-UGM-1 (2018); SGG-UGM-2 (2020). These models were downloaded via IAG's International Centre for Global Earth Models, ICGEM (http://icgem.gfz-potsdam.de/ICGEM/). These models were tested against three gravity field functions, such as absolute gravity, Bouguer gravity anomaly and geoid undulation. Two study areas of Sri Lanka; Jaffna and Bandarawela, representing flat and rugged terrains with recently observed gravity data were used for validation of the global models against observed gravity and Bouguer anomalies. The published GPS-levelling heights of the FBMs of Sri Lanka were used for the geoid undulation test. Figure 2 shows the computational procedure of this analysis.

RESULTS AND DISCUSSION

The ground gravity observations were made using a Scintrex CG-6 auto gravity meter. 171 ground gravity observations in Bandarawela and 267 observations in Jaffna were used for the analysis. SRTM global DEM with 1 arc second resolution was used for terrain correction and computation of Bouguer gravity anomalies. Figure 3 shows the locations of observed gravity stations and the variation of Bouguer gravity anomalies in both regions. Bandarawela is a hilly area in Sri Lanka, and as expected, Bouguer anomalies were negative, ranging from -3 to -33 m Gal. According to Heatherton et al. (1975), Jaffna is a negative anomaly region, and this current analysis also showed negative anomalies which were ranging from -17 to -62 m Gal.

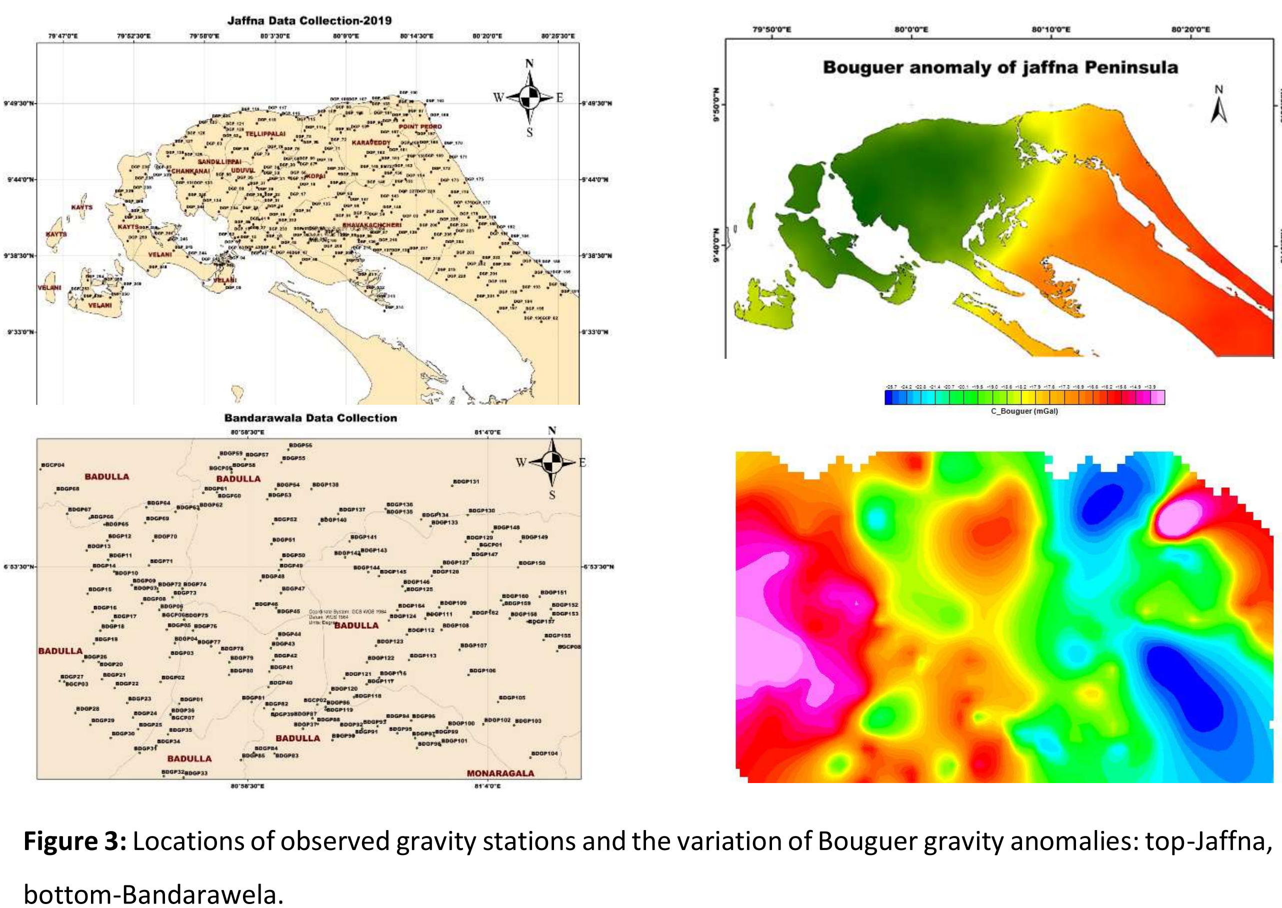

Figure 3: Locations of observed gravity stations and the variation of Bouguer gravity anomalies: top-Jaffna, bottom-Bandarawela.

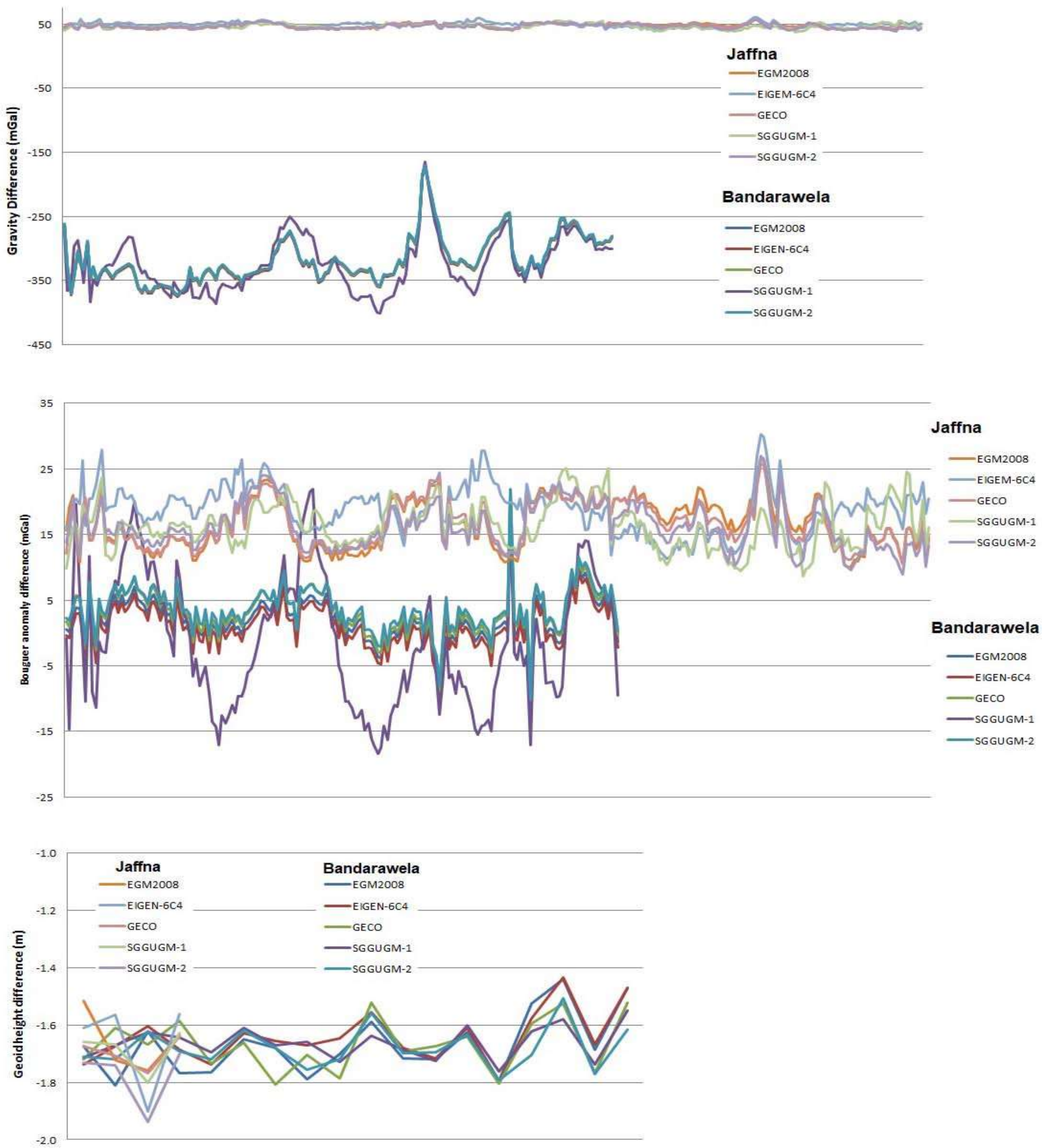

Figure 4: variations between observed and model gravity functional (Gravity, Bouguer anomaly and geoid height)

Figure 4 shows the variation of the difference between observed and model gravity field quantities. According to the figure, a smooth variation can be seen in Jaffna and rapid fluctuations can be observed in Bandarawela. The reason is topography. In Jaffna, no rugged topography, so that gravity varies smoothly, in contrast, Bandarawela is a mountainous area, so the topography variations reflect in gravity.

For geoid height comparison, 4 GPS benchmarks in Jaffna and 18 in Bandarawela were used. Unlike in gravity, both regions show around -1.6m mean difference between the observed and model derived values, reflecting the bias between the Sri Lankan GPS-levelling datum and the global geoid. Table 1 shows the simple statistics of the difference between observed and model values.

Table 1: Statistics of the difference between model and observed gravity field functional

Bandarawela						
Model (degree and order 2159)	Gravity difference (mGal)		Bouguer anomaly difference (mGal)		Geoid height difference (m)	
	Mean	STD	Mean	STD	Mean	STD
EGM2008	-317.744	13.459	2.006	1.346	-1.668	0.145
EIGEN-6C4	-318.705	13.501	1.047	1.387	-1.641	0.188
GECO	-316.624	13.538	3.118	1.425	-1.667	0.138
SGGUGM-1	-323.965	19.085	-1.28	6.158	-1.662	0.115
SGGUGM-2	-316.096	13.455	3.647	1.342	-1.678	0.067
Jaffna						
Model (degree and order 2159)	Gravity difference (mGal)		Bouguer anomaly difference (mGal)		Geoid height difference (m)	
	Mean	STD	Mean	STD	Mean	STD
EGM2008	47.109	0.815	16.719	0.853	-1.656	0.083
EIGEN-6C4	49.355	3.101	18.957	3.003	-1.659	0.035
GECO	47.058	1.969	16.673	1.889	-1.696	0.023
SGGUGM-1	46.494	4.54	16.321	4.487	-1.689	0.021
SGGUGM-2	46.563	0.305	16.202	0.397	-1.777	0.021

The large mean difference of gravity shows the omission errors of the model values, it happens due to the truncation of the model to their maximum degree and order. This implicitly represents that even high-resolution models are not capable enough to represent the actual gravity. It is high in rugged mountainous regions as numerically shown in Bandarawela and low in Jaffna.

The positive Bouguer anomaly difference in Jaffna shows that the model values are higher than the observed values. This is expected and happened due to the surface subsurface negative mass anomalies in Jaffna Peninsula that represent a sedimentary basin filled with Miocene limestone (Tantirigoda and Geekiyanage, 1988).

Unlike in gravity, the mean geoid height difference is more or less common for both regions. It is around -1.6m. The large mean and comparatively small STD shows the clear bias of the geoid height difference. This implicitly shows the inconsistencies of the Sri Lankan GPS-levelling datum.

As a whole, according to the statistics, SGGUGM-2 shows a better agreement with all three gravity field functional in terms of standard deviations.

CONCLUSIONS

In this present study, five HR-GGMs are evaluated against the ground gravity and GPS-levelling data. Two regions with flat and rugged terrain were utilized to see their variations properly. Analysis of gravity and Bouguer anomaly revealed that the global models are not capable of representing the gravity features in rough mountainous areas but fit quite well with flat terrain. A clear bias of Sri Lankan GPS-levelling datum can be seen in the results of geoid height analysis of high mean value. Overall, the recently released SGGUGM-2 model shows a better agreement with ground gravity data in Sri Lanka.

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